University of Mannheim
School of Social Sciences
Math Refresher for Political Science, Fall 2025
Carlos Gueiros

Problem Set: Set Theory I

1. Let $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8\}$ and $C = \{6, 8\}$. Find following:
   (a) $A \cup B$
   (b) $A \cap B$
   (c) $A \cap B^C$
   (d) $B - A$
   (e) $C - B$
   (f) $A \cap C$
2. Let $A = \{a, b, c, d\}$, $B = \{1, 2, 3, 4\}$ and $C = \{a, b, 1, 2\}$. Show that:
   (a) Distributivity: $(A \cap C) \cup (B \cap C) = (A \cup B) \cap C$
   (b) Associativity: $(A \cap B) \cap C = A \cap (B \cap C)$
   (c) De Morgan Laws: $C - (A \cup B) = (C - A) \cap (C - B)$
3. Determine which of the following formulas are true. If any formula is false, find a counterexample to demonstrate this using a Venn diagram.
   (a) $A \setminus B = B \setminus A$
   (b) $A \subseteq B \iff A \cap B = A$
   (c) $A \cup B = A \cup C \implies B = C$
   (d) $A \subseteq B \iff A \cup B = B$
   (e) $A \cap B = A \cap C \implies B = C$
   (f) $A \setminus (B \setminus C) = (A \setminus B) \setminus C$
4. Explain in words why it is true that for any sets A, B, C:
   (a) $(A \cup B) \cup C = A \cup (B \cup C)$
   (b) $(A \cap B) \cap C = A \cap (B \cap C)$
   (c) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   (d) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

5. Find the interior point(s) and the boundary points(s) of the set $\{x : 1 \leq x \leq 5\}$.

6. Why does every set in $\mathbb{R}$ that is nonempty, closed, and bounded have a greatest member?

7. Which of the following sets are open, closed, or neither?

   (a) $D = \{x \in \mathbb{R}^1 : x = 2 \text{ or } 3 < x < 4\}$
   (b) $A = \{(x, y) \in \mathbb{R}^2 : x^2 \leq y \leq 1\}$
   (c) $B = \{(x, y) \in \mathbb{R}^2 : x^2 < y < 1\}$
   (d) $C = \{(x, y) \in \mathbb{R}^2 : x^2 \leq y < 1\}$
   (e) universal set

8. Sketch the following functions:

   (a) $f(x) = 2$
   (b) $f(x) = 3x - 1$
   (c) $f(x) = x^2 + 2x + 1$
   (d) $f(x) = (x - 3)^{-1}$
   (e) $f(x) = |2x - 2|$
   (f) $f(x) = e^{2x}$
   (g) $f(x) = -\sqrt{x}$

9. Which of the following functions is injective, bijective, or surjective?

   (a) $a(x) = 2x + 1$
   (b) $b(x) = x^2$
   (c) $c(x) = \ln x$
   (d) $d(x) = e^x$